Hypothesis testing (`v0`)

Remark. In this exercise sheet, we are going to work with small sample sizes to avoid long and boring computations. In practice however,

- all computation are done from your computer (sorry we were allowed only 3 slots for Labs
- we work with (much) larger sample sizes

Exercise 1 (✔☰). A pharmacist prepares a drug that can be lethal if it contains too much of a certain toxic substance. Consider the hypothesis H_0: *the drug does not contain more of the toxic substance than the authorized dosage.*

a) Give the alternative hypothesis H_1.

b) Which kind of error the pharmacist should minimize: Type I (α) or Type II (β). Justify.

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Exercise 2 (✔☰). In a production process, the proportion of defective parts produced is known to be 0.05. The process is periodically monitored by taking a sample of size 20 and examining the produced parts. If 2 or more out of the 20 parts are defective, the process is stopped and considered out of control.

a) Give the null and alternative hypothesis.

b) Compute the probability α of doing a Type I error.

c) Give and sketch the power of the test when the probability of defaulting is 0.06, 0.08, 0.1, 0.15, 0.2 and 0.25.

d) Compare your answers to questions b) and c) when the process is now considered out of control when 3 or more parts are defective.

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Exercise 3. In the 1970s, female athletes from East Germany were renowned for their astonishing athletic performances. The Olympic ethics committee at the time, questioning these remarkable achievements, enlisted the services of Dr. Volker Fischbach. He selected nine female German athletes and conducted analyses measuring the amount of virilizing hormonal substances (known as androgens) per liter of blood. The results are as follows:

$$3.22 \quad 3.07 \quad 3.17 \quad 2.91 \quad 3.40 \quad 3.58 \quad 3.23 \quad 3.11 \quad 3.62$$

The hypothesis is H_0: *the East German athletes did not dope.*

Based on a normal model and knowing that the average amount of androgens in a woman is $\mu_0 = 3.1$, did Dr. Fischbach conclude that the East German athletes doped?

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Exercise 4 (✓☰). The eye and hair color of 95 individuals were recorded, yielding the following results:

	Light hair	Dark hair
Blue eyes	32	12
Brown eyes	14	22
Other	6	9

Is hair color independent of eye color ($\alpha = 5\%$) ?

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Exercise 5. A new scale model (N) is being tested. The manufacturer claims it is more precise than the old model (A). A series of 15 objects (the same for both scales) are weighed on each scale. The results are:

$$A\colon 5.25, 11.0, 8.47, 12.5, 2.60, 10.2, 10.1, 8.27, 10.4, 8.06, 12.3, 9.74, 8.94, 10.6, 13.3$$
$$N\colon 9.97, 8.50, 6.06, 9.41, 9.29, 9.87, 8.98, 10.1, 9.18, 10.5, 11.8, 8.76, 10.3, 9.85, 7.97$$

Perform a test to determine whether the new scale is indeed more precise than the old one ($\alpha = 5\%$).

Numerical data : $\sum x_A = 141.7$, $\sum x_A^2 = 1446$, $\sum x_N = 140.5$ et $\sum x_N^2 = 1340$.

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Exercise 6 (✓☰). Does jogging reduce heart rate?

To answer this question, eight non-joggers participated in a running program. Their heart rates were measured before and after the program. The results are as follows:

Avant:	74,	86,	98,	102,	78,	84,	79,	70
Après:	70,	85,	90,	110,	71,	80,	69,	74

Perform a hypothesis test at the $\alpha = 5\%$ significance level to determine whether jogging significantly reduces heart rate. Clearly state the null hypothesisq H_0 and the alternative hypothesis H_1. Justify the choice of the test used.

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Exercise 7. The melting point of 16 samples of a brand of vegetable oil was measured, yielding $\overline{x} = 94.32$. We assume that the distribution of the melting point follows a Normal distribution with $\sigma = 1.20$.

a) Test $H_0 : \mu = 95$ against $H_1 : \mu < 95$ at a significance level $\alpha = 0.01$.

b) If the true value of μ is 94, what is the power of the test?

c) What sample size n is required to ensure that $\beta(94) = 0.1$ when $\alpha = 0.01$?

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Exercise 8. An experiment was conducted to compare the influence of two dietary supplements on chicken growth. To do this, chicks were randomly divided into two groups.

- Group A: Chicks were fed soybean meal.

- Group B: Chicks were fed fava beans.

After six weeks, the weight (in grams) of each chicken was measured:

A: 243, 230, 248, 327, 329, 250, 193, 271, 316, 267, 199, 171, 158, 248

B: 179, 160, 136, 227, 217, 168, 108, 124, 143, 140

Perform a hypothesis test ($\alpha = 5\%$) to determine whether there is a significant difference between the effects of these two diets. Assume that the data follow a normal distribution and that the variances of both groups are equal but unknown.

Numerical data: $\sum x_A = 3450$, $\sum x_A^2 = 888268$, $\sum x_B = 1602$, $\sum x_B^2 = 270068$

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